

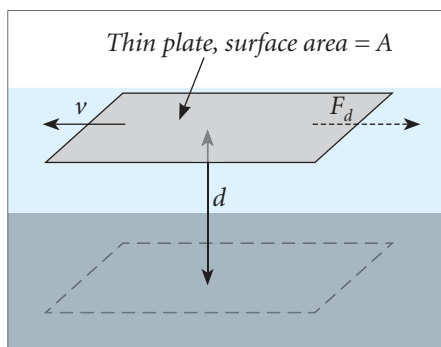
Viscosity: the mathematical treatment

viscosity 1. c. In scientific use, the tendency of a liquid or gas to resist by internal friction the relative motion of its molecules and hence any change of shape; the magnitude of this, as measured by the force per unit area resisting a flow in which parallel layers unit distance apart have unit speed relative to one another; also called *absolute* or *dynamic viscosity*; *kinematic viscosity*, the dynamic viscosity divided by the density of the fluid. — *Oxford English Dictionary*

VISCOSITY is a measure of the resistance of a fluid to flow. This resistance arises from the attractive forces between the molecules of the fluid. A fluid will only flow if enough energy is supplied to overcome these forces.

For a body to be able to move through a fluid, the fluid has to flow around or across it. Therefore, the energy required to move a body through a fluid is directly related to the degree to which that fluid resists flow, *i.e.* its viscosity.

Because physicists visualise a body as “cutting” its way through a fluid, the word “shear” is generally used to mean “move”. The English physicist Sir Isaac Newton (1642-1727) defined the viscosity of a fluid at a given temperature in terms of the resistive (drag) force F_d experienced by a thin plate of surface area A shearing



through the fluid with velocity v at a distance d from a reference surface (*e.g.* the wall of the container):

$$F_d = A\mu(v/d)$$

In this expression, “ μ ” (the Greek letter mu) represents the viscosity of the fluid. Another commonly-used symbol for viscosity, particularly in the UK, is “ η ” (the Greek letter eta).

The quantity v/d (the velocity divided

by the distance to the reference surface) is called the “velocity gradient” or “shear rate”. It is generally symbolised by “ $\dot{\gamma}$ ” (“gamma-dot”). Using this symbol, and dividing both sides of the above equation by A , we get

$$F_d/A = \mu\dot{\gamma}$$

The quantity F_d/A (the force divided by the surface area) is often called the “shear stress”, symbolised by σ (the Greek letter sigma). A little substitution and rearrangement gives us the following definition of viscosity:

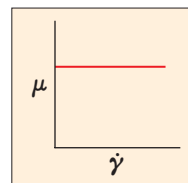
$$\mu = \sigma/\dot{\gamma}$$

I.e. the viscosity of a fluid is equal to the *shear stress* divided by the *shear rate*.

Non-Newtonian fluids

Newton knew that viscosity changed with temperature. He also assumed that viscosity was always independent of shear rate, *i.e.* the viscosity would remain the same no matter how quickly the plate was shearing through the fluid.

Fluids which exhibit this type of behaviour are called “Newtonian”. If we measure the viscosity at a number of shear rates for such a fluid and plot the results on a graph, we will see a flat straight line:



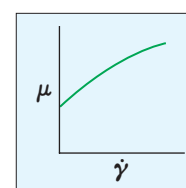
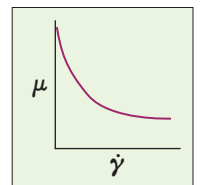
Water, alcohol and thin motor oil are typical Newtonian fluids. Many fluids, however, are “non-Newtonian” — at a given temperature, the viscosity depends on the shear rate and/or the length of time during which the fluid is subjected to shear.

There are several types of non-Newtonian fluid. They are classified

on the basis of the way in which their viscosities change:

(i) *Pseudoplastic (shear-thinning)*: the viscosity falls as the shear rate increases.

Examples: paints, shampoo.

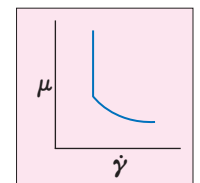


(ii) *Dilatant (shear-thickening)*: viscosity rises as the shear rate increases.

Example: mixtures of sand and water.

(iii) *Plastic (Bingham)*: behave as a solid until a certain force (the “yield value”) is applied, after which they may display either Newtonian or non-Newtonian flow characteristics.

Example: ketchup (catsup).



Thixotropy and rheopexy

The viscosity of some fluids changes over time even if the shear rate remains constant. If the viscosity falls, the fluid is said to be “thixotropic”; if it increases, the fluid is said to be “rheopexic”. Both thixotropy and rheopexy may occur together with other flow characteristics, or only at certain shear rates.

Some liquids are time-thinning as a result of a breakdown in structure. This phenomenon is called “*rheomaiaxis*”.

While rheopexic fluids are rare, thixotropic fluids are very common. The best-known examples are paints and heavy printing inks. 